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The area of an n-gon inscribed in the unit circle is given by the equation $A = n\sin(\frac{180^o}{n})\cos(\frac{180^o}{n})$. Since $180^o = \pi$ radians, $A = n\sin(\frac{\pi}{n})\cos(\frac{\pi}{n})$. To prove that the sequence consisting of the areas of consecutive n-gons inscribed in the unit circle converges to π as $n \to \infty$ we will evaluate $n \overset{\lim}{\to} \infty\ A$.
Since $A = n\sin(\frac{\pi}{n})\cos(\frac{\pi}{n})$, $n \overset{\lim}{\to} \infty\ A$=$n \overset{\lim}{\to} \infty\ [n\sin(\frac{\pi}{n})\cos(\frac{\pi}{n})]$.
Since $\sin(x)\cos(y) = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$,
$\sin(\frac{\pi}{n})\cos(\frac{\pi}{n}) = \frac{1}{2}(\sin(\frac{2\pi}{n}) + \sin(0)) = \frac{1}{2}\sin(\frac{2\pi}{n})$.
Thus $n \overset{\lim}{\to} \infty\ [n\sin(\frac{\pi}{n})\cos(\frac{\pi}{n})]$ =$n \overset{\lim}{\to} \infty\ [\frac{n}{2}\sin(\frac{2\pi}{n})] = \frac{1}{2}\ n \overset{\lim}{\to} \infty\ \frac{\sin(2\pi n^{-1})}{n^{-1}}$.
Since $n \overset{\lim}{\to} \infty \sin(2\pi n^{-1}) = 0$ and $n \overset{\lim}{\to} \infty\ n^{-1} = 0$ L'Hopital's Rule applies.
Thus $\frac{1}{2}\ n \overset{\lim}{\to} \infty\ \frac{\sin(2\pi n^{-1})}{n^{-1}} = \frac{1}{2}\ n \overset{\lim}{\to} \infty\ \frac{\cos(2\pi n^{-1})(-2\pi n^{-2})}{-n^{-2}}$
$= \frac{1}{2}\ n \overset{\lim}{\to} \infty\ [\frac{-2\pi n^{-2}}{-n^{-2}}\cos(2\pi n^{-1})] = \pi\ n \overset{\lim}{\to} \infty \cos(\frac{2\pi}{n}) = \pi$
Therefore $n \overset{\lim}{\to} \infty\ A = \pi$